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Probability of detection – the approach to combine probabilistic fracture mechanics with NDT – where we are ?

Prawdopodobieństwo detekcji - podejście do łączenia probabilistycznej mechaniki pękania z NDT - gdzie jesteśmy?

ABSTRACT

The contribution presents the detail discussion on various problematic aspects of utilization of both fracture mechanics based methodology and NDT technology to conduct the assessment of failure of components. Consequently, it shows a software tool concerning Probability of Detection – Probability of Sizing Concept, which allow to bring both methodologies (Fracture Mechanics and NDT) together and to meet in a joint approach. The contribution introduces in the application of one software approach, which allows modeling and simulation of real scenarios in detail, based on a variety of properties of different relevant materials from practice.

Keywords: *fracture mechanics,* assessment of failure, NDT

STRESZCZENIE

Praca ta przedstawia szczegółową dyskusję na temat różnych aspektów problematyki jednoczesnego wykorzystania metodyki opartej na mechanice pękania i technologii badań nieniszczących (NDT) do przeprowadzania oceny i predykcji uszkodzenia komponentów. W konsekwencji, przedstawiono oprogramowanie narzędziowe odnoszące się do koncepcji prawdopodobieństwa wykrycia (Probability of Detection– Probability of Sizing), które pozwalają na połączenie obu metodologii (mechaniki pękania i NDT) i działanie w ramach wspólnego podejścia. Praca przedstawia implementację w ramach oprogramowania podejścia, które umożliwia modelowanie i szczegółową symulację rzeczywistych sytuacji, przy uwzględnianiu wielu właściwości różnych materiałów używanych w praktyce.

Słowa Kluczowe: *mechanika pękania, predykcja uszkodzenia, NDT*

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1. Introduction

Based – for instance - on standards like BS 7910: 2005, Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures, fracture mechanics experts have introduced a methodology to assess failure of components taking into account the geometry of the component and the flaw, the applied mechanical loading, including residual stresses, and material properties. In case of simple geometries, according to canonical co-ordinates, like cylindrical pipes, by analytical solutions the Stress-Intensity-Factor (SIF) can simply be calculated – so far the NDT technology has delivered reliable information to the relevant flaw size/geometry. In case of complicated geometries, numerical approaches like FE, BE or FD have to be used. However, in order to evaluate, under which conditions, a component due to loading, fails or not, to answer this question, needs – besides the knowledge of the actual mechanical stress distribution – the knowledge of material properties. These are the Critical Stress-Intensity-Factor, K_{IC}, also named fracture toughness, the yield Rp0.2 and the tensile strength σ_{UTS} of the material of interest, which have to be known. It is trivial, these three properties are not constant as function of position in a component, they are statistically distributed. The distribution functions, with mean values and standard deviation, depend on a manifold of influence parameters. They depend basically on the material, but also its properties changes due to manufacturing and machining (casting, forging, welding, …) and ageing in service life (fatigue, creep, corrosion, irradiation, …). Therefore, the use of constant material parameters, taken from tables in text books, can be reliable only, if worst case considerations are asked for, which always are extremely conservative. When realistic approaches are the objective of a critical engineering assessment then the statistical distributions have to be taken into account, which only realistically can be determined due to a high number of destructive tests, i.e. with a tremendous financial effort. That is why most of component producers do not follow this expensive approach as nearly all consensus standards, not yet are asking for such a procedure. Only in nuclear technology we have, with the ASME Lower Bound Fracture Toughness Curve, respectively, the Master Curve Concept, first important and relevant steps into stochastic scenarios.

The effort is much more enhanced if the fact is observed, that all of our NDT techniques suffer under measurement uncertainties, also following probabilistic, and not, deterministic laws. The tool, to bring both methodologies (Fracture Mechanics and NDT) together, to meet in a joint approach, is the Probability of Detection – Probability of Sizing Concept which is discussed in the

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here presented contribution, based on probabilistic Monte Carlo simulation. Modeling is also the only chance to reliably reduces the costs by simulation to play realistic scenarios. The contribution introduces in the application of one software approach, which stands, as one example, for many others available on the market, and, which allows this modeling in detail, based on a variety of properties of different relevant materials from practice.

Fracture mechanics as part of theoretical mechanics and especially elasticity theory started with developments early in the 19th century combined with names of scientists like Kolosov 1908/1909 [1], Ingles 1913 [2], Griffith 1921 [3], Westergaard 1939 [4], Sneddon 1946 [5], Irvin 1949 and 1957 [6,7], Dugdale 1960 [8], Burdekin and Stone 1966 [9], Rice 1968 [10], Heald, Spink, and Worthington, 1972 [11], Dowling and Townley, 1975 [12], and, Wallin, 1991 [13].

Kolosov [1], in the former USSR, was the first, who has used the calculus of complex-valued, so-called analytical, functions to describe on their basis elasticity problems. These functions fulfil the Cauchy-Riemann differential equations which implicitly ask for their Real- and Imaginary Function parts, to be real valued potential function, i.e., fulfilling Laplace' equation. The calculus, later by Westergaard [4] was further developed to the theory of the complex-valued stress functions $\Phi(x1, x2)$, which are the generalized tool, to find at that time analytical solutions describing the stress enhancement in the vicinity of cracks, simply modeled as ellipses, assumed in elastic, plane, thin sheets under boundary loads in the infinite. The stress functions Φ are solutions of the Bi-Harmonic Differential equation:

$$\Delta\Delta\Phi = 0 \tag{1}$$

what can be verified by calculation, when certain important presumptions are fulfilled, which are:

The balance of moments and forces in a volume element and in the thermal equilibrium: $\sum F_{x_1} = \sum F_{x_2} \equiv 0$ (balance of forces); $\tau_{x1,x2} = \tau_{x2,x1}$ (balance of moments) which results in:

$$\frac{\partial}{\partial_{x_1}}\sigma_{x_1} + \frac{\partial}{\partial_{x_2}}\tau_{x_1x_2} = 0 \;\; and \;\; \frac{\partial}{\partial_{x_2}}\sigma_{x_2} + \frac{\partial}{\partial_{x_1}}\tau_{x_1x_2} = 0 \tag{2}$$

Hooke' law as materials law (under plane strain or plane stress conditions)

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix} = \frac{E}{1-\upsilon^2}\begin{pmatrix}1 & \upsilon & 0\\ \upsilon & 1 & 0\\ 0 & 0 & \frac{1-\upsilon}{2}\end{pmatrix}\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{pmatrix}$$

or

$$\begin{pmatrix}\varepsilon_{11}\\ \varepsilon_{22}\\ 2\varepsilon_{12}\end{pmatrix} = \frac{1}{E}\begin{pmatrix}1 & -\upsilon & 0\\ -\upsilon & 1 & 0\\ 0 & 0 & 2(1+\upsilon)\end{pmatrix}\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \sigma_{12}\end{pmatrix} \tag{3}$$

E is Young' module and ν is Poisson' ratio, and the so-called Compatibility Conditions which are the mathematical, necessary and sufficient conditions that the existence of a unique deformation or strain field in an elastic body is guaranteed, when the body is exposed to a continuous single valued displacement field.

In 2-D plane strain problems, which here are of interest, the strain-displacement relations are:

$$\varepsilon_{11} = \frac{\partial}{\partial x_1}u_1 \quad \varepsilon_{22} = \frac{\partial}{\partial x_2}u_2 \quad \varepsilon_{12} = \frac{1}{2}\left[\frac{\partial}{\partial x_2}u_1 + \frac{\partial}{\partial x_1}u_1\right] \tag{4}$$

which, by further differentiation, results in the compatibility condition (5).

$$\frac{\partial^2}{\partial x_2{}^2}\varepsilon_{11} - 2\frac{\partial^2}{\partial x_1 \partial x_2}\varepsilon_{12} + \frac{\partial^2}{\partial x_1{}^2}\varepsilon_{22} = 0 \tag{5}$$

From (5) follows that only a plane displacement field $u(x_1, x_2)$ is compatible with a plane strain field, and by calculation, with (2) and Hooke' law (3), $u(x_1, x_2)$ is a solution of (1).

It was Sneddon [5], who, based on Westergaard' complex-valued stress function in plane polar coordinates, translated the origin of the coordinate system, in the position of the crack tip, and developed an approximation in the case, that the distance r from the tip is small, compared with the half crack length a ($r << a$). In his formulas (6) a typical factor occur ($\sigma\cdot\sqrt{\pi a}$). By comparing the manifold of existing analytical solutions in cases of degenerating the ellipses to spheres [1] or infinite small slits [3], [6], Irvin [7] was able to show, that all of them, when investigated near the crack tip, show this factor, which he called Similarity Parameter. This parameter is nothing else, as the Stress Intensity Factor (SIF), which was firstly introduced in use, with his approach and terminology. Furthermore, the mathematical singularity in case of r = 0 at the crack-tip position, was found in each of these solutions.

$$\begin{pmatrix}\sigma_{11}\\ \sigma_{22}\\ \tau_{12}\end{pmatrix} = \frac{\sigma\sqrt{\pi a}}{\sqrt{2\pi r}}\begin{pmatrix}\left[1-\sin\frac{\Theta}{2}\sin\frac{3\Theta}{2}\right]\cos\frac{\Theta}{2}\\ \left[1+\sin\frac{\Theta}{2}\sin\frac{3\Theta}{2}\right]\cos\frac{\Theta}{2}\\ \left[\sin\frac{\Theta}{2}\cos\frac{\Theta}{2}\cos\frac{3\Theta}{2}\right]\end{pmatrix} \tag{6}$$

This singularity really cannot exist, because a real material answers with plastic deformation if the stress is larger than the yield strength. This is true, in the case of linear elastic (brittle) materials too, even if the plastic zone here is very small, restricted immediately on the crack-tip vicinity.

It was Griffith, who found, that failure loads observed at brittle materials were tremendous smaller than predicted by theories describing the separation of atomic bonds. In other words: on one hand, there are additionally to the observed individual crack, numerous other microscopic imperfections in the material. And, on the other hand, a certain amount of stored potential energy is consumed by another effect than only driving a crack to failure. Griffith' conclusion, based on the law of energy preservation, was, that crack propagation can only happen if the amount of potential energy, stored in the specimen, which is released by crack propagation, is larger than the amount of energy, which is consumed by increasing the crack surface. If the external loads are fixed, they do not perform any work. The change of the potential energy is therefore the decrease of the stored elastic energy in the body. Griffith found an excellent agreement of his approach, in case of brittle material, i.e. glass.

Griffith' result was ignored up to the work of Irvin [6, 7] in the early 1950s because most of the used material was metal and ductile and surface energy increase, predicted by Griffith, was too high. Even more, the mathematical singularity at the crack tip was not real, as the materials have developed a plastic zone.

Irvin discussed the total thermodynamic energy G dissipated due to crack growth, which is the sum of increasing of the surface energy (2γ; factor 2 because of the two surfaces in the crack, γ-surface energy [σ 2π·a]/E'; E'=Young' modulus E in case of plane stress- and $E'=E/(1-\nu^2)$ under plane strain-conditions) and an energy part consumed by the ductile material to increase the plastic zone, named G_p.

In the case of the important Crack-Opening-I-mode of fracture and linear elastic materials (neglecting G_p), he found the relationship:

$$G = K_I^2 / E' \quad (7)$$

With this approach Irvin found also the experimental access to estimate the Critical Stress Intensity Factor KIC under plane strain conditions in the linear elastic fracture mechanics.

Dougdale [8], with his strip yield model, then in his approach, taking the plastic zone into account, by virtually enlarging the geometric crack-length a to $a + a_p$, where a_p has the dimension of the plastic zone in x-direction. According his approach, in the stress function, the mathematical singularity at $y = 0$ and $x = a$ should be compensated by the new introduced singularity at $y = 0$ and $x = a + a_p$. Then the material, macroscopically, acts again linear elastic. Following this idea, he was able to describe the size of the plastic zone approximately to:

$$a_p = \pi/8 \times (K_I / Y_p)^2,\ Y_p - \text{Yield strength} \quad (8)$$

According to Burdekin' and Stone' approach [9], the measurement of δ, the CTOD (Crack-Tip-Opening-Displacement) -technique was introduced, to estimate the Critical Stress-Intensity-Factor, i.e. the fracture toughness:

$$CTOD = \delta = K_I^2 / (E\, Y_p) \quad (9)$$

The procedure, how to measure CTOD in an incremental-stress/load-increase test up to the critical-one, by using a clip-gage (knife-edge-strain gage) and calculate from the measured crack-mouth opening the crack-tip-opening in case of the critical load, was regulated by defining standard testing procedures and test specimens of which the geometry has to observe restrictions, because of the geometry dependence.

Based on the approach of Rice [10] the so called J-Integral was introduced, as he found in case of elastic plastic materials (non-linear materials) a generalized energy release approach. Based on the yield strip model stress function he found a Path-Independent-Energy Integral, called J, of which the calculation, nowadays, is part of every actually available fracture mechanics software. In the case of linear elastic material consistently J is in conformance with G of (7); in the elastic-plastic (non-linear) case, a relationship to the CTOD is found:

$$J = Y_p \times CTOD \quad (10)$$

Heald, Spink and Worthington [11] analyzed the displacement v in y-direction at the crack-tip behind a plastically deformed material and its contribution to a crack-opening $\delta = 2v$.

They found:

$$\delta = 2v = 8 \frac{a}{\pi E} Y_p \times \ln\left(\sec\left[\frac{\pi \sigma}{2 Y_p}\right]\right);\ \sec(x) \equiv 1/\cos(x) \quad (11)$$

By introducing an Effective Stress Intensity Factor $K_{eff} = \sqrt{(J\,E)}$ and taking into account that $J = (Y_p\ \delta)$, then, by simple calculation, follows:

$$K_{eff} = \sqrt{\left\{\frac{8\, a\, Y_p^2}{\pi} \times \ln\left(\sec\left[\frac{\pi \sigma}{2 Y_p}\right]\right)\right\}} = Y_p \sqrt{\pi\, a} \times \sqrt{\frac{8}{\pi^2} \times \ln\left(\sec\left[\frac{\pi \sigma}{2 Y_p}\right]\right)} \quad (12)$$

To account for real structural components, the Yield strength is replaced by the strength for plastic collapse, identified with the flow stress $\sigma_{flow} = (Y_p + \sigma_{UTS})/2$. The ratios K_r and S_r are defined:

$$Kr = K_I / K_{eff} \quad \text{and} \quad Sr = \sigma / \sigma_{flow} \quad (13)$$

By calculation follows:

$$K_r = S_r \frac{1}{\sqrt{\frac{8}{\pi^2} \times \ln\left(\sec\left[\frac{\pi}{2} S_r\right]\right)}} \quad (14)$$

the so-called 2-Criteria Approach, which defines the FAD, the Failure-Assessment Diagram, on which we concentrate in the next chapters and which at first was introduced by Dowling and Townley in 1975 [12].

Series of codes, guidelines and standards exist describing the technical rules to destructively determining the relevant material properties. Only the most important should be mentioned here:

- ASTM E1820, 2009 – Standard Test Method for Measurement of Fracture Toughness. Determination of J, CTOD, and K,
- ASTM E1921, - Test Method for Determination of Reference Temperature T0 for Ferritic Steels in the Transition Region,
- BS 7448 Part 1, 1991 – Fracture Mechanics Toughness Tests, Methods for Determination of KIc, critical CTOD and J-values for metallic Materials,
- BS 7448 Part 2, 1997 – similar as Part 1, but for Metallic Welds,
- BS 7910, 2005 – Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures,
- API 579, 2007 – Recommended Practice for Fitness for Service, 2nd edition.

A most modern approach, to characterize the brittle-to-ductile transition of martensitic/bainitic steels used in pressure vessels and pipes in nuclear power plants, goes back to K. Wallin [13], who found an engineering procedure, to describe the statistical scatter in the J-Integral if specimens are tested as function of temperature, by assuming a Weibull Statistic. However, the technique delivers no access to the toughness value, but to the transition temperature T_0.

2. PVrisk

This chapter is an introduction in the software PVrisk [14] developed theoretically by D. Cioclov and compiled as software by J. Kurz in the Fraunhofer Institute for NDT in Saarbrücken, Germany. The software is an example, for comparable software products available in the market. However, a special feature is given by the integration of randomness, concerning the geometrical parameters of a crack, i.e., its crack size can scatter according to statistical distributions, as well as the randomness can be in the material data. The randomness is introduced due to Monte Carlo simulation. This fact allows the calculation of the POF, i.e., the probability of failure. An absolute new and a unique feature in such type of software is the introduction of the use of Non-destructive Testing (NDT). This is done by introducing the concept of the POD, i.e., the probability of detection, a terminology which goes back to the NASA in US and W. Rummel as a pioneer [15]. In PVrisk theoretically assumed POD-models can be applied, to virtually study the advantage of POD, but, POD-curves based on experimentally determined NDT result can be implemented after data fitting [16, 17] into PVrisk too.

2.1 The principle of Failure Assessment

The assessment of the failure risk of a load-carrying structural component is based on the analysis of the component state, taking into account the initial strength of the material and the production technology used in the manufacturing process, as well as the types of flaws which may be pre-existent or generated during operation. The local states of the material, its microstructure and the state of stress/strain in the regions with irregularities have to be taken into account. The analysis of these regions must consider the applied NDT and especially its capacity to distinguish between a crack- and a no-crack-signal.

For metals, in the case of a deterministic approach, this ends with a prediction either of failure or non-failure [18, 19]. The method was first developed for such deterministic analyses and is a recognized tool for failure analysis and in many standards available today, as mentioned in chapter 1. However, realistic discussions make consideration of the reliability of the data investigation necessary. This succeeds, if the statistical fuzziness or uncertainty is integrated into the failure assessment by the use of Monte Carlo simulations, based on probability distribution functions. The result of the probabilistic analysis is, therefore, a statement about the probability of failure (POF) of which – so far the consequences are known – the risk of failure can be calculated. Figure 1 shows a FAD (Strip Yield Model), as one of two possible, which can be used in PVrisk.

The abscissae Sr and ordinates Kr are introduced in accordance with formulae (15) and (16):

$$K_r = K_I / K_{IC} \quad (15)$$

$$S_r = \sigma_{ref} / \sigma_f \quad \sigma_f = (\sigma_y + \sigma_{UTS}) / 2 \quad (16)$$

Here, K_I is the stress intensity factor, K_{1C} is the fracture toughness of the material and σ_{ref} is the reference stress, which can be a superposition of the membrane stress σ^m_{ref}, a bending stress σ^b_{ref} and a residual stress Q. σ_y is the yield strength ($R_{p0.2}$, 0.2% yield limit) and σ_{UTS} is the tensile strength (R_m).

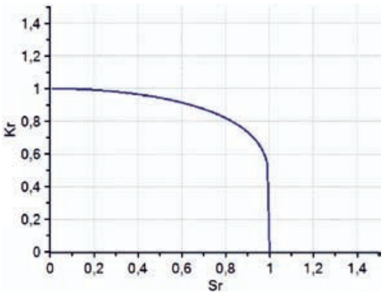

Fig. 1. Failure assessment diagram (FAD) – the failure bounding curve is shown
Rys. 1. Krzywa zniszczenia FAD (Failure assessment diagram)

The stress intensity factor (SIF) is made up of two parts. The first part is influenced by the load-induced stresses (primary stresses) and the second part takes residual stresses (secondary stresses) into account (Equations (17) to (19)):

$$K_I = K_I^p + K_I^s \quad (17)$$

$$K_I^p = (F_{SIF}^m \times \kappa^m \times \sigma^m_{ref} + F_{SIF}^b \times \kappa^b \times \sigma^b_{ref}) \times \sqrt{(\pi b)} \quad (18)$$

$$K_I^s = F_{SIF}^m \times Q_m + F_{SIF}^b \times Q_b) \times \sqrt{(\pi b)} \quad (19)$$

Here, F_{SIF}^m is the geometry-dependent correction factor (see also as example equation (6)), taking the membrane loads into account, and F_{SIF}^b is the corresponding correction factor when bending loads are applied. Weld seams can be accounted for, through the correction factors κm (membrane loads) and κb (bending loads), as well as through the residual stresses (Q_m, Q_b). The b in the term √(πb) is the crack size – in the geometry module, the crack depth b.

In the FAD, the analysis is based on obtaining a value-pair (S_r, K_r). Here, K_r is a characteristic for the brittleness and S_r for the ductility of a material state situation. The bounding curve (failure curve) shown in Figure 1 is derived by elastic-plastic theoretical considerations according to [20], where, in Equation (20), Sr is varied in the interval [0, 1] to find, respectively, the K_r values:

$$Kr = Sr / \sqrt{\{(8/\pi 2) \ln [\sec[(\pi/2) Sr]\}} \quad (20)$$

Development of the formula see equations (11) to (14).

Failure assessment can be provided by evaluation of the position of the point pair (S_r, K_r) in the FAD. A position in the area formed by the bounding curve and the coordinate axes in the FAD obviously indicates no failure. If the point pair lies on the bounding curve, or is outside of it, a failure occur.

2.2 Geometry Module

The model of a semi-elliptical crack of length 2a, with a = 10 mm and depth b = 5 mm, is chosen here for the crack geometries in the following, oriented in the axial direction and precisely positioned symmetrically in the middle position of the cylinder, assumed with length 2L = 2000 mm, thickness t = 20 mm and outer radius R_0 = 200 mm. Several, different crack models are implemented into PVrisk. The actual model has the advantage that a semi-analytic formulae exist, to calculate the stress intensity factor (SIF). If the crack geometry is assumed to be distributed according to a statistical distribution function, this also has to be defined in the geometry module. In the case of crack-like defects, the length and depth values can be normally distributed, log normally distributed or distributed according to a Weibull distribution.

It should be mentioned here, that extensive destructive statistical investigations must be performed for a given component geometry, in order to determine the distribution (frequency) of defect sizes, which, for instance, remain in the component after production. In order to achieve a reliable statistical result, usually some thousand metallographic micrographs must be produced and examined. Only then the type of distribution function can reliably be determined with mathematical methods. Early publications in the field, historically speaking, have already studied this problem [21, 22]. Here, the probabilities of different NDT techniques detecting surface-breaking cracks have already been discussed. Even if an automated NDT technique is applied [22], where the human factor influence is low, the uncertainty in the NDT performance is not only a problem when the crack size is small and near the physical limit of detectability. Larger defects can also be missed by NDT (“the evidence of absence is not the absence of evidence” [23]).

2.3 Material Module

The material properties must be defined in the material module. These are the values as shown in Figure 2 for the yield strength, the tensile strength and the fracture toughness as fixed values in the case of a deterministic evaluation. This module is also the input window for the statistical parameters, if a probabilistic approach is performed (middle part of Figure 3, left-hand side, here a normal, Gauss distribution). Distribution functions according to a normal distribution, a log normal distribution or a Weibull distribution can be selected. However, as already mentioned in the previous paragraph - when we discussed the case of the crack geometry distribution functions - a high number of destructive tests must also be performed here, in order to predict the type of the material parameter distribution and its statistical scatter.

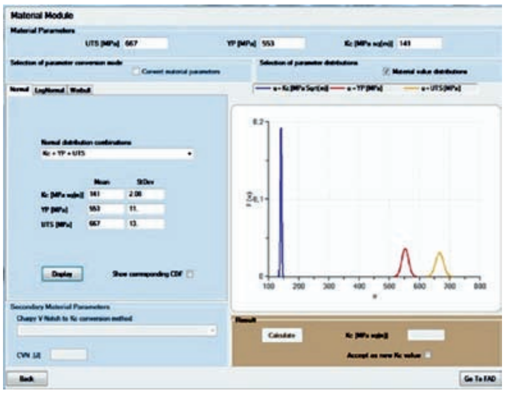

Fig. 2. The material module, screenshot of the GUI
Rys. 2. Widok interfejsu GUI modułu materiałowego

The background to discuss here a material state, called <recovery annealed>, reflects the fact, that this material, by heat treatment at the last applied stress relieve annealing temperature, can be fully recovered, bringing the Cu particles again in solid solution.

Tab. 1. The steel WB36 in the states <recovery annealed> and <thermally aged>
Tab. 1. Stal WB36 w stanach <wyżarzanie wtórne> i <starzenie termiczne>

microstructure	Yp in MPa	UT in MPa	KIc = Kc in MPa √m
<recovery annealed>	553±11	667±13	141±2.08
<thermally aged> at 350 °C for 57.000 h	661±13.22	776±15.52	96±36

In the case-studies discussed later, the copper alloyed steel 15 NiCuMo Nb 5 (WB36) which is applied for piping and vessels in German power plants is studied in detail [24]. This material is prone for thermal ageing due to Cu-precipitates in the nanometer range, when exposed at service temperatures of 350 °C and higher, for a duration, as usual within one service period, of 57.000 h. In the <thermal aged> state, a shift of the ductile to brittle temperature (Charpy test) of about 70 °C can be observed. Table 1 summarizes the material properties in the microstructure states <recovery annealed> and <thermally aged>.

So far only values of Charpy energy for toughness are available (Fig. 3), these values can be converted in PVrisk by using model-based assumptions into a fracture toughness value, here, of KIc = 141 MPa√m (in the software abbreviated with Kc). In the destructive examinations [24] mentioned above, the scatter of the data in terms of a standard deviation in case of the <recovery annealed material> was very small in the range of ±2 %. These values were used in the following sensitivity analyses. Both deterministic and probabilistic approaches were applied. In the case of the probabilistic assessment, 106 trials (oppertunities) were assumed for the Monte Carlo simulation. The load is assumed as inner pressure when the cylinder is closed at both ends.

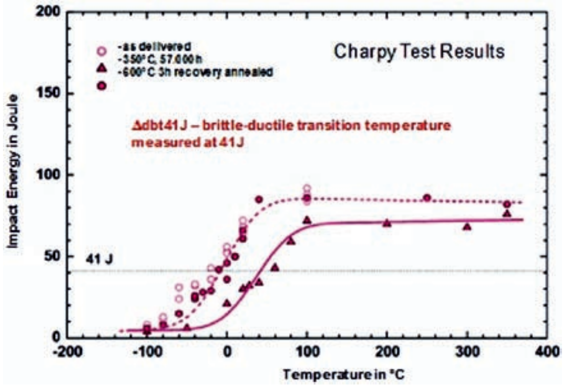

Fig. 3. Charpy Test Results of WB 36
Rys. 3. Wyniki testowania młotem Charpy'ego stali WB36

2.4 Deterministic approach

Table 2 documents the development of the load (inner pressure) to failure, in both of the material microstructures if a, the half crack length, is increasing as well as the crack depth b. The thermally aged material with degradation shows lower crack size values to failure, i.e. when the limiting curve is reached. However, the differences are not very large. It seems to be, that the higher strengths values compensate the lower toughness.

Tab. 2. The steel WB36 in the states <recovery annealed> and <thermally aged>
Tab. 2. Stal WB36 w stanach <wyżarzanie wtórne> i <starzenie termiczne>

microstructure	Half critical crack length a in mm	Critical crack depth b in mm	Stress to failure in MPa
<recovery annealed>	23	18	17.5
<thermally aged> at 350 °C for 57.000 h	22	17	17.7

Tab. 3. The steel WB36 in the state <thermally aged>, but reduced wall thickness
Tab. 3. Stal WB36 w stanie <starzenie termiczne>, przy zredukowanej grubości ściany

Critically half crack length a in mm	Critically crack depth b in mm	Stress applied in MPa	Wall thickness in mm
22.2	17.0	17.5	20
17.4	12.1	17.5	15
12.5	7.2	17.5	10

A question was asked: What happens, if we reduce the wall thickness from 20 mm to 15 mm and then 10 mm ? Table 3 give the answer.

We learn, the material fails earlier, i.e., smaller critical crack sizes are obtained.

If we continuously now increase the pressure, the critical crack size to failure is further reduced. Table 4 documents the result.

Tab. 4. The steel WB36 in the state <thermally aged>, but with enhanced pressure
Tab. 4. Stal WB36 w stanie <starzenie termiczne>, przy zwiększonym ciśnieniu

Critically half crack length a in mm	Critically crack depth b in mm	Wall thickness t in mm	Internal pressure in MPa
12.5	7.2	10	17.5
12	6.5	10	20
11.2	5.7	10	23
10.9	5.4	10	24
10.7	5.1	10	25

2.5 Probabilistic approach

PVrisk allows using mixed modes (deterministic and probabilistic parameters) for analyzing different scenarios [24, 25]. So, for instance, the crack geometry can be taken as deterministically given (a = 10 mm, b = 5 mm, wall thickness t = 10 mm) and the material parameters are allowed to vary statistically, for instance according to a Gauss distribution as shown in Figure 2. In Table 5 the result in case of the <thermally aged> microstructure is presented:

Tab. 5. POF in 10^6 trials, the scatter in toughness as influence parameter was reduced
Tab. 5. POF po 10^6 próbach, rozrzut w twardości jako parametru wpływu został zredukowany

	POF – Probability of Failure in 10^6 trials	Pressure in MPa – primary circuit service pressure in a Nuclear PWR	Scatter KIc in MPa × √m
Microstructure <thermally aged> at 350 °C for 57.000 h	8.3×10^{-2}	15	96 ± 36
	7.8×10^{-2}	15	96 ± 35.2
	7.5×10^{-2}	15	96 ± 34.4
	6×10^{-2}	15	96 ± 32
	2×10^{-2}	15	96 ± 24
	7×10^{-3}	15	96 ± 20
	1×10^{-3}	15	96 ± 16
	2.9×10^{-5}	15	96 ± 12
	5×10^{-6}	15	96 ± 11
	0 - safe	15	96 ± 8

We learn, that the reduction of the scattering in toughness has a tremendous influence on the POF.

The NDT concept, by using models for the probability of detection (POD), will be discussed in the following. A quantitative consideration of NDT methods in assessment procedures requires quantitative statements about the reliability of the applied NDT method. The determination of the probability of detection (POD) is one possibility to quantify the probability to detect a specific flaw with a NDT method. Details about the POD concept can be found in [16, 17]. The POD is expressed in form of a cumulative distribution function. This concept is represented by different usable models within PVrisk. As well as for material parameters, realistic scattering values should be taken also for POD results from evaluation of real cracks in their geometrical sizes. The differences between results gained with real and artificial cracks can be exemplarily found in [26]. If no data is available, conservative assumptions should be taken, for a first run.

PVrisk allows simulating the POD in different models representing different approaches to map the detectability. We will discuss here only one of these models, the use of an asymptotic power law, according to Cioclov [27] (Figure 4).

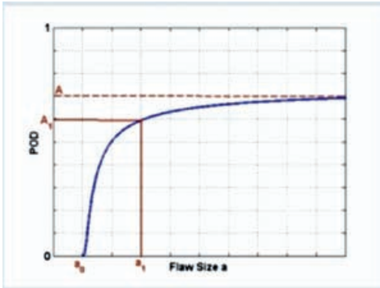

Fig. 4. Assymptotic power law to model the POD
Rys. 4. Asymptota modelu krzywej POD

In Table 6 the parameters A, A1, a0 and a1 are defined, describing the model and its quality to reliably detect a crack-like defect. By reducing the values of a0 and a1, as well as by increasing the value of A1, the POD is improved. In the aerospace industry, based on the damage tolerance design principle, the parameter a1 has a special meaning. The value here is called a90/95 and gives the defect size, for instance the crack length of a crack penetrating the wall of the airplane hull [28], at which the POD reaches 90% with 95% confidence.

Tab. 6. POD models according to an asymptotic power law
Tab. 6. Modele POD

POD model	A	A_1	a_0 in mm	a_1 in mm
POD1	1	0.9	2	3
POD2	1	0.9	1	2
POD3	1	0.9	1	1.5
POD4	1		0.5	1

In Table 7 the result of the application of the different POD models is discussed. The material is the <thermally aged> WB36 with a Gauss distributed crack geometry amean =10 mm and bmean = 5 mm, and scatter in the data Δa = 2 mm, Δb = 1 mm, wall thickness t = 10 mm, internal pressure p = 15 MPa. The material properties scatter according the Gauss distributions of Figure 2.

Tab. 7. Material properties and crack sizes are Gauss distributed
Tab. 7. Właściwości materiału i rozmiary wady o rozkładzie Gaussa

POD model	Number of failures	POF Probability of Failure
No POD	7100 in 10^{+6}	7.1×10^{-3}
POD1	195 in 10^{+6}	1.95×10^{-4}
POD2	132 in 10^{+6}	1.32×10^{-4}
POD3	127 in 10^{+6}	1.27×10^{-4}
POD4	60 in 10^{+6}	6×10^{-5}

Obviously, an enhanced POD reduces the number of failures. However, a further realistic enhancement in reduction of the POF can only be achieved, not by enhancing POD, but, if and only if, the mean values of a and b are reduced, for instance, to amean = 8 mm, bmean = 2 mm, and the scatter to Δa = 1 mm, Δb = 0.5 mm. Then a POF-value POF = 2 ×10-5 can be obtained, i.e., only a number of 20 remaining failures in 106 opportunities.

3. Conclusion

Obviously, PVrisk is an elegant tool to simulate inspection trials of NDT. As the software allows deterministic as well as probabilistic approaches, a wide parameter study can be performed taking into account the variation of geometrical and material data. As the most sensitive parameter concerning the probability for failure, the fracture toughness absolute (deterministic approach) or mean value (probabilistic approach) as well as its scatter (standard deviation) can be identified.

As the fracture toughness values are a result of destructive tests more of these tests have to be performed by steel as well as component producers to evaluate the scatter in these data for each really safety relevant component form and applied steel grade, and not, to utilize values "sometimes" determined in laboratory and published in tables. Whereas the prediction of strength properties at some real production goods by NDT in steel industry, more and more, becomes to be the today state of the art [29], this is not yet the case with fracture toughness. Therefore, it has to be the objective of material characterization as a NDT task in the future to develop correlations between NDT quantities and toughness properties, in order to predict at the component locally toughness by NDT in in-service inspection strategies.

The sensitivity analysis here performed was based on material properties values, taken at ambient temperature, and not at elevated service temperatures, where toughness is in the upper shelf regime and much higher. Therefore, the results are extremely conservative, However, the critical crack sizes here obtained as result of the studies are no real problem to be detected by enhanced NDT. The inner pressure, in most of the simulations have been assumed higher than service pressure. The conclusion is: Our components are designed with high safety margins.

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5. References

[1] Kolosov G. V., On One Application of the Theory of Functions of Complex Variables to the Plane Problem of the, Mathematical Theory of Elasticity; Yurev, 1909, Complete Dictionary of Scientific Biography, 2008
[2] Inglis C. E., Trans Inst Nav Archit, 1913, Vol 55, pp 219
[3] Griffith A. A., The phenomena of rupture and flow in solids, Phys. Trans. Roy. Soc., A series, Vol 221, 1921, pp 163-198
[4] Westergaard H. M., Bearing pressures and cracks, Journ. Applied Mech. 6, 1939, pp 49-53
[5] Sneddon I. N., The distribution of stress in the neighborhood of a crack in an elastic solid, free download from http://rspa.royalsocietypublishing.org/, Proc. Roy. Soc A series 187, original 1946
[6] Irvin G. R., Fracturing of metals, ASM, Ohio, 1949, p 147
[7] Irvin G. R., Analysis of stresses and strains near the end of a crack transversing a plate, J. Appl. Mech, 24, 1957, pp 361-374
[8] Dugdale D. S., Yielding of a steel containing slits, J. Mech. Phys. Solids. 18, 1960
[9] Burdekin F. M., Stone D. E. W., The crack opening displacement approach to fracture mechanics in yielding materials, J. of Strain Anal. 1, 1966, pp 145-153
[10] Rice J. R., A path-independent integral and the approximate analysis of strain concentration by notches and cracks, J. of App. Mech. 35, 1968, pp 379-386
[11] Heald P.T., Spink, G.M., and Worthington, P.J.: Post Yield Fracture Mechanics, Mater. Sci. and Eng., 10, 1972, pp. 129-138
[12] Dowling A.R., Townley C.H.A.: The effect of defects on structural failure: A two criteria approach, J. Press. Vess. and Pip., V3, 1975, pp. 77-107
[13] Wallin K., Statistical modeling of fracture in the ductile-to-brittle transition range, defect assessment in components, fundamentals and applications, Mech. Eng. Publ., 1991, pp 415-445
[14] PVrisk, User Manual of Version: Jochen H. Kurz and Dragos Cioclov, Version 1.1, 18. June 2008
[15] Rummel W.D. et al.: Application of NDE Reliability to Systems, ASM Handbook, Vol 17, NDE and Quality Control 3rd edition, ASM International, 1994, pp 674-688
[16] ASTM E3023–15: Standard Practice for Probability of Detection Analysis for â Versus a Data
[17] Annis, C.: MIL-HDBK-1823A. Nondestructive Evaluation System Reliability Assessment. Department of Defense Handbook, Wright-Patterson AFB, USA 2009
[18] API RP 579: American Petroleum Inst., Fitness for Service, 2000
[19] Berens, A.P.: NDE Reliability Data Analysis, ASM Handbook V. 17, NDE and Quality Control 3rd ed., ASM, 1994, pp 689-701
[20] Dougdale, D.S.: Yielding of Steel Sheets, J. Mech. Phys. Solids, 8, 1960, pp 100-104
[21] Kremer, K.J.: The Therm-O-Matic procedure, a new procedure for the online examination of steel products concerning surface defects, Report volume 4 to the 3rd European Conference on NDT, Florence, AIPND, 15-18 October 1984.
[22] Palanisamy, R., Morris, C.J., Keener, D.M., and Curran, M.N.: On the accuracy of AC flux leakage, eddy current, EMAT and ultrasonic methods of measuring surface connected flaws in seamless steel tubing, Review of Progress in Quantitative Nondestructive Evaluation, Plenum Press, Volume 5A, pp 215-223, 1986.
[23] http://en.wikipedia.org/wiki/Evidence_of_absence
[24] Kurz, J.H., Cioclov, D., and Dobmann, G.: On the integrating of non-destructive testing and probabilistic fracture mechanics, In: C. A. Graubner and H. Schmidt (eds), 6th International Probabilistic Workshop, Technische Universität Darmstadt, 2008, pp 43-56
[25] Dobmann, G., Cioclov, D., and Kurz, J.H.: The role of probabilistic approaches in NDT defect detection, classification and sizing, Welding in the World, 51, 5/6, 2007, pp 9-13
[26] Kurz, J.H., Jüngert, A., Dugan, S., Dobmann, G., Boller, Ch.: Reliability considerations of NDT by probability of detection (POD) determination using ultrasound phased array. Engineering failure analysis 35, 2013, pp.609-617
[27] Dobmann, G, Ciclov, D, and Kurz, J.H.: NDT and fracture mechanics. How can we improve failure assessment by NDT - Where we are – where do we go?, Insight Vol 53 No 12 December 2011
[28] Schmidt, H.J., Schmidt-Brandecker, B., and Tober, G.: Design of modern aircraft structure and the role of NDT, Proceedings of the 7th European Conference on NDT, Copenhagen, May 1998
[29] Wolter, B., and Dobmann, G.: Micromagnetic testing for rolled steel, Proceedings of the European Conference on Non-Destructive Testing, 9, Th.3.7.1, 2006